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A STUDY OF PARTICLE MOTION INDUCED BY TWO-DIMENSIONAL LIQUID OSCILLATIONS

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Abstract—The motion of single particles in liquids undergoing two-dimensional oscillations has been studied both experimentally and theoretically. By imposing combined vertical and horizontal oscillations on a liquid, particle mean motion can be controlled so that individual particles either rise against gravity or fall. The one dimensional equation of particle motion was modified to suit this situation and numerical solutions were used to predict the occurrence of the rising motion although agreement with experiment was only qualitative. A dimensional analysis approach combined with experiments allowed prediction of particle behaviour within the range of variables of this study. Extension of the single particle studies to mixtures of quartz and galena in various flowing liquids allowed prediction of the conditions under which separation and upgrading of minerals could occur and this was verified experimentally.

INTRODUCTION

The mean motion of a particle passing through a liquid is altered by the presence of any imposed oscillations on the liquid, unless slow, or creeping motion of the fluid occurs throughout the relative motion. For higher Reynolds numbers (particle $Re > 0.1$), it has been reported by Houghton (1963) and subsequently many authors (Schöneborn 1975) that uniform vertical oscillations imposed on a fluid can retard the free fall under the influence of gravity. Van Oeveren & Houghton (1971) further showed that for non-uniform oscillations, particles could be caused to rise against gravitational forces. A situation similar to these non-uniform oscillations occured if the fluid was simultaneously oscillated in the horizontal and vertical directions. This has been studied both experimentally and by analysis to determine the degree of control available over individual particle motion, and to isolate the parameters which most affect the mean motion.

The results of these single particle studies were used to predict the behaviour of slurries containing concentrations of different density particles when subjected to two-dimensional oscillations. This led to an experimental programme in which slurries were continuously fed into a separation chamber to demonstrate that particle separation can be achieved and to determine the limits of application of the single particle studies.

SINGLE PARTICLE STUDIES

Theory

A well proven theoretical analysis does not exist for describing the high Reynolds number motion of particles in accelerated liquid systems (Herringe 1976; Schoneborn 1975). The common approach has been to replace the Stokes drag force term which appears in the one-dimensional equation of particle motion by the steady-state drag force in terms of the drag coefficient. For a spherical particle of diameter, d, density ρ_p in a fluid of viscosity μ and density ρ_f, its velocity $\mathbf{U}_p$ at time t can be determined from

$$\frac{\pi d^3}{6}\cdot\rho_p\frac{d\mathbf{U}_p}{dt} = -C_d\cdot\frac{\pi d^2}{8}\rho_f(\mathbf{U}_p-\mathbf{U}_f)|\mathbf{U}_p-\mathbf{U}_f| + \frac{\pi d^3}{6}\rho_f\frac{d\mathbf{U}_f}{dt} + \frac{\pi d^3}{6}(\rho_f-\rho_p)g + \frac{1}{2}\cdot\frac{\pi d^3}{6}\rho_f\frac{d}{dt}(\mathbf{U}_f-\mathbf{U}_p) + \frac{3}{2}d^2\sqrt{(\pi\mu\rho_f)}\int_{t_0}^{t}(t-t')^{-1/2}\frac{d}{dt}(\mathbf{U}_f-\mathbf{U}_p)\,dt'. \qquad [1]$$

The imposed fluid motion is described by the fluid velocity $\mathbf{U}_f$. The coefficient C_d is usually taken to be the steady-state drag coefficient and g represents the gravitational acceleration.

The validity of the last term (the so-called Basset term) for two-dimensional motion is doubtful since it accounts for the affects of previous motion in the one-dimensional case, and even then it may not be valid for higher Reynolds number flows. In any case, its inclusion in a numerical analysis requires cumbersome iterations (Herringe 1976) so for the purposes of this initial investigation it was not included in the solutions which follow. If [1] (neglecting the history term) is assumed to apply in the instantaneous direction of motion, the result is a pair of ordinary differential equations which can be solved by simple numerical methods to predict particle motion. Such solutions have been obtained and compared to the experimental results.

Dimensional analysis

From the equation of particle motion, and from intuitive reasoning, the variables which contribute to the mean particle velocity U_p can be combined through dimensional analysis to give

$$\frac{\bar{\mathbf{U}}_p}{U_t} = f_1\left(\frac{\rho_p}{\rho_f}, \frac{\mu}{\rho_f \omega_V A_V^2}, \frac{A_V \omega_V^2}{g}, \frac{A_V}{d}, \frac{A_V}{A_H}, \frac{\omega_V}{\omega_H}, \tau\right). \qquad [2]$$

This expression assumes the imposed fluid motion is comprised of two simple harmonic components in the horizontal (subscript H) and vertical (subscript V) directions, of angular frequency ω and amplitude A with the relative phasing represented by phase angle τ. The particle fall velocity under steady-state conditions U_t was introduced so that the effect that the oscillations have on the motion can be readily gauged.

In the series of experiments undertaken in this paper, frequency and amplitude ratios were held constant at $\omega_V/\omega_H = 2$ and $A_V/A_H = 0.25$. These values were chosen so that the maximum values of the liquid accelerations in the vertical and horizontal directions were the same. Further, it was thought that higher frequency ratios (and hence lower amplitude ratios) would have added little to the asymmetry of the vertical component of the motion and hence the particle motion would have been qualitatively similar to the results reported here. The dimensionless velocity ratio is thus a function of 5 variables

$$\frac{\mathbf{U}_p}{U_t} = f_2\left(\frac{\rho_p}{\rho_f}, \frac{\mu}{\rho_f \omega_V A_V^2}, \frac{A_V \omega_V^2}{g}, \frac{A_V}{d}, \tau\right)$$
$$= f_2(\phi, St, G, \beta, \tau). \qquad [3]$$

These variables are referred to as the density ratio ϕ, Stokes number St, acceleration number G, amplitudes scale β, and phase angle, τ.

Experimental

A cylindrical chamber, 110 mm high and 82.5 mm diameter, and containing various glycerol–water solutions was oscillated by the mechanical vibrator shown schematically in figure 1. The vibrator was operated at frequencies up to 112 Hz in the vertical direction with amplitudes up to 3 mm, giving maximum fluid accelerations up to 77 g for these tests. The viscosities of the various solutions ranged from 0.9 to 21 cp and were determined experimentally for each sample fluid (other than water) with a Brookfield rotational viscometer. Individual glass particles were photographed to indicate the shape and size and only those appearing spherical were chosen for the tests. The glass particles and steel ball bearings were separately dropped from the particle injector and their trajectories recorded by viewing through a window 85 × 60 mm with a camera at speeds up to 4000 frames per sec. This allowed the particle trajectories to be plotted from the film record and the mean velocities to be determined. For the fixed values of amplitude ratio and frequency ratio, the path travelled by the container and the fluid could be altered by the relative phasing between the two driving cams. In figure 2, several paths have been drawn, indicating the zero reference position and the variation of path with phase angle. The zero position was chosen as that corresponding to bottom dead centre of the driving cams, and the phase angle refers to rotation of the horizontal driving cam from this position.

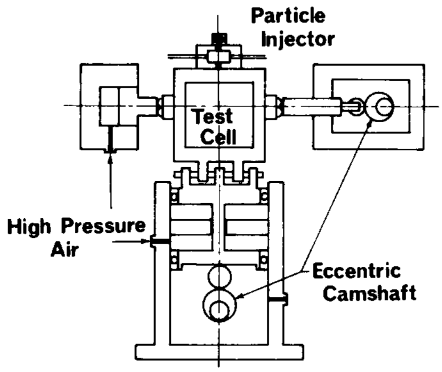

Figure 1. Mechanical vibrator for fluid oscillations.

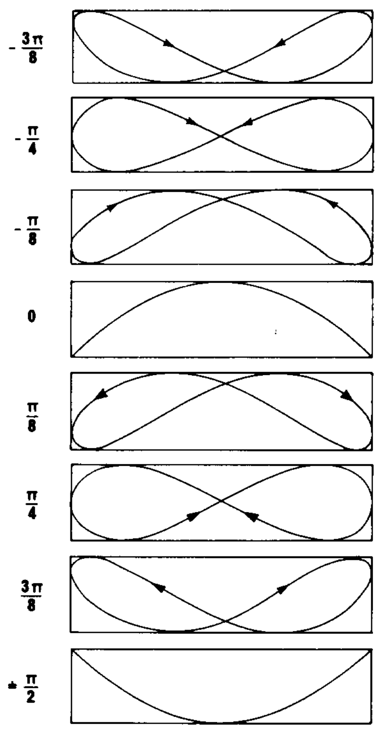

Figure 2. Fluid motion for varying phase angles between vertical and horizontal motions.

Discussion of experimental results

To verify that the functional relationship of [3] includes all of the relevant variables, two sets of experiments were conducted for different frequencies, amplitudes, viscosities and particle size, but with similar values for the dimensionless parameters. It was initially thought that fluid oscillations would have most effect on the particle motion for higher accelerations. The experimental conditions chosen thus allowed maximum acceleration within the limitations of the experimental rig. The particle sizes were chosen as the most suitable for high speed photography. In figure 3, the two sets of results which show the variation of velocity ratio with phase angle have been compared, and show similar dependence on phase angle. In each case, the particle steady-state terminal velocities were determined from published drag coefficients rather than by experiment, and for the test labelled B two different particles of nominally the same diameter were used. As shown in table 1, there are small differences in the dimensionless parameters for the two sets of results, but in spite of this, velocity ratios are generally within ± 0.15 of the mean line drawn in figure 3. The most interesting feature of these results is the strong rising tendency of

Table 1. Experimental variables and dimensionless parameters for single particle studies

	Experimental conditions						Dimensionless variables			
Run	d (mm)	ρ_p (kg/m³ × 10⁻³)	ρ_f	μ (cp)	$\frac{\omega_v}{2\pi}$ (sec⁻¹)	Av (mm)	G	β	St × 10³	ϕ
A	0.53	2.96	1.116	5.0	112	1.0	50.5	1.89	6.38	2.65
B	1.68	2.96	1.179	24.8	68	3.0	54.2	1.78	5.52	2.21
C	1.59	7.80	1.178	24.8	68	3.0	54.2	1.89	5.52	6.62
D	1.68	2.96	1.00	1.0	68	3.0	54.2	1.89	0.263	2.96
E	1.59	7.80	1.00	1.0	68	3.0	54.2	1.89	0.263	7.80
F	0.80	2.96	1.0	1.0	68	3.0	54.2	3.75	0.263	2.96
G	0.80	7.80	1.0	1.0	68	3.0	54.2	3.75	0.263	7.80
H	0.54	2.96	1.0	1.0	28	1.0	3.2	1.85	5.68	2.96
I	0.54	2.96	1.0	1.0	68	1.0	18.6	1.85	2.38	2.96
J	0.54	2.96	1.0	1.0	94	1.0	35.6	1.85	1.69	2.96

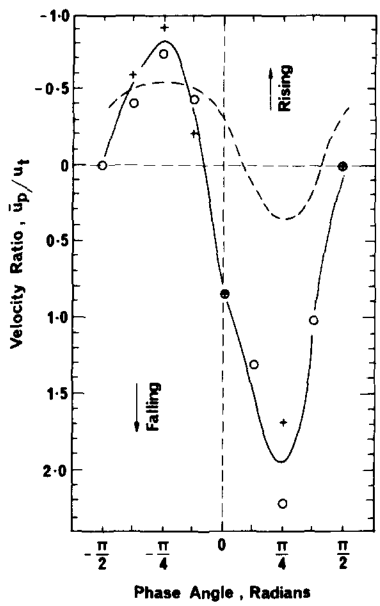

Figure 3. Variation of velocity ratio with phase angle for dimensionally similar condition. +—Expt A, O—Expt B, —theoretical.

particles for the negative angles, in magnitude up to 80% of the value of steady-state fall velocity.

Also included in figure 3 are the velocity ratios predicted by the numerical simulation, showing a qualitative agreement with experiments and predicting the rising tendency of the particles. It is not surprising that closer agreement was not obtained since [1] has been shown by both Herringe (1976) and Schöneberg (1975) to be inadequate for one-dimensional motion, particularly in oscillating flow situations where there is a suppression of vortex shedding.

With the validity of the relationship in [3] established, and the phase angle dependence demonstrated in figure 3, each of the parameters was varied in further experiments. To determine the effect of density ratios, the experiments of curve B were repeated with steel spheres instead of glass. The comparison of mean velocity ratios may be seen in figure 4. At the phase angle of $-3\pi/8$, the relative velocity between a glass and steel particle would be approx. 20 cm/sec, or almost twice the difference in their terminal settling velocities (11.6 cm/sec) for this fluid. It seems that this is a possible condition for separating particles with these two density ratios, especially since the lighter particles actually rise through the fluid for these conditions. Also included in figure 4 are the theoretical predictions for these conditions which once again show qualitative agreement with experiment, but could not be used to predict a separation condition.

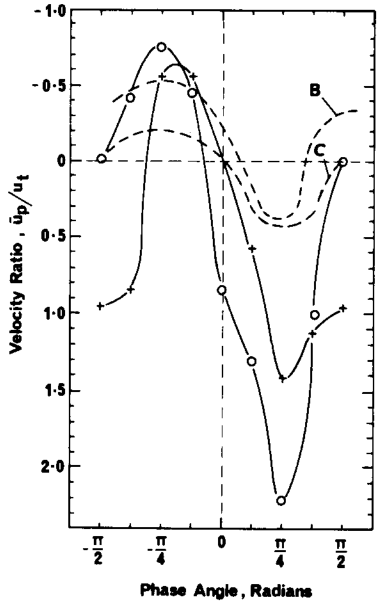

Figure 4. Velocity ratio variation with phase angle for different density ratios. O—Expt *B*, +—Expt *C*, -----theoretical.

The second dimensionless parameter in [3] is referred to as the Stokes number. Experiments to study the effect of the Stokes number on particle motion were carried out by varying the liquid medium (from aqueous glycerol to water) and repeating the experiments of figure 4. A comparison of the two sets of results is given in figure 5 which shows that although the Stokes number has been reduced from 5.52×10^{-3} to 0.263×10^{-3} (a factor of 21) the phase angle dependence has not greatly altered. However, the maximum relative velocities (at $\tau=-3\pi/8$) between the two different density materials was only increased by about 26% above the steady-state difference for the lower Stokes number while it was almost doubled for the other tests.

As a simple means of looking at effects of amplitude scale (A_V/d) on the particle motion, particles of different size were tested (other parameters are independent of particle size). The results are given in figure 6 where the velocity ratios for the smaller glass particles were more

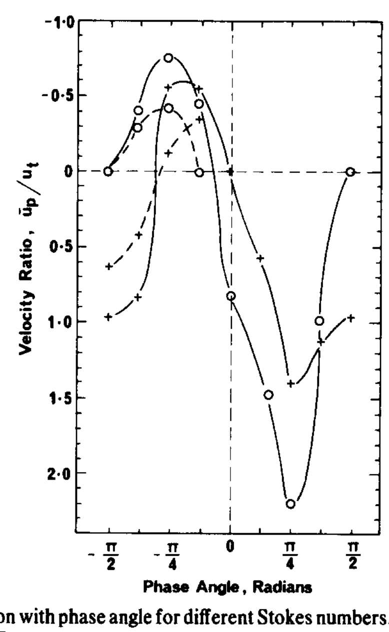

Figure 5. Velocity ratio variation with phase angle for different Stokes numbers, and for two density ratios. —O—Expt. *B*; —+—Expt. *C*; --O-- Expt. *D*; --+-- Expt. *E*.

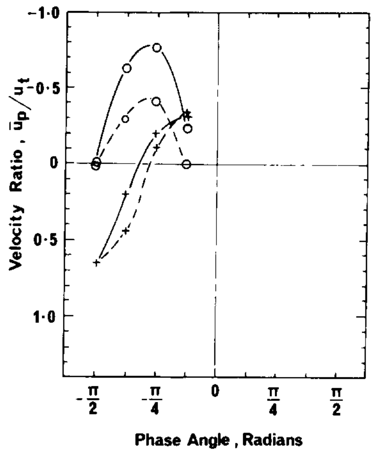

Figure 6. Velocity ratio variation for different amplitude scale conditions. --O-- Expt. D; --+-- Expt. E; —O— Expt. F; —+— Expt. G.

than doubled while for the steel particles the change was not as significant. The enhancement of relative velocities between particles for the phase angle of $-3\pi/8$ was similar for both values of amplitude scale since for the 1.6 mm particles, the relative velocity was 26% above their relative steady-state velocity while for the 0.8 mm particles the relative velocity was increased by 28%. The availability of suitable particles and the minimum resolution possible with the high speed photography limited the range of amplitude scales, although the range was extended significantly in the continuous flow studies to be discussed.

To determine the effect of acceleration number on particle motion, experiments with similar values for the dimensionless parameters as those in figure 3 were carried out. The experiments were with 0.54 mm diameter glass particles in water at various frequencies (and thus G) with a vertical amplitude of 1 mm (similar to tests labelled A). The results are presented in figure 7. For the experiments at 28 Hz (indicated by J on figure 7), all dimensionless parameters were similar to those of figure 3 except for the acceleration number which was 3.2 instead of about 50. The acceleration number was increased by increasing the frequency of motion and this altered the value of the Stokes number since it is also a function of frequency. However, if the water results of figure 7 are extrapolated to a value of $G = 50$–55, then the velocity ratio will be about -0.6 to -0.7 and this compares favourably with the corresponding value of -0.72 in figure 3. This suggests that variation of the Stokes number within the range of these experiments (from $St \times 10^{+3} = 1.69$–6.38) does not significantly alter the results so that the results of figure 7 can be considered to apply for similar conditions with only the acceleration number varied. These

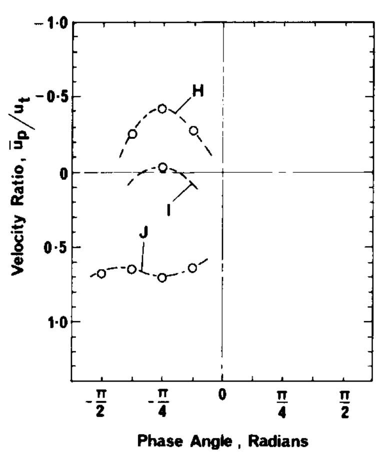

Figure 7. Effect of variation of acceleration number on rising motion of glass particles. Letters refer to experimental conditions in Table 1.

results indicate that further increases in the acceleration number will have a decreasing effect on the particle rise velocity.

It should be noted that the acceleration number is not the maximum absolute fluid acceleration during a cycle, but only the maximum of the vertical component. For the phase angle of $-3\pi/8$ for example, the absolute maximum acceleration is approx. 1.36 times the maximum vertical acceleration. A potentially useful outcome of this study is the understanding of the way in which particle mean velocity, in terms of direction and magnitude, can be controlled by imposing oscillations on the fluid. As a possible application of this result, preliminary tests have been conducted with concentrations of particles in a continuous flow situation, with the aim of separating grains of different density minerals. The single particle studies have indicated a limited range of parameters over which such a separation seemed possible, but they were limited by the minimum particle size which could be followed photographically and by the features of the driving mechanisms.

CONTINUOUS FLOW STUDIES

Experimental

These studies involved feeding quantities of particles as a slurry into a test cell with two separate outlets. The essential features of the flow passage of the test cell are that it was a narrow diverging passage (4 mm wide) with baffles to reduce gross circulation within the cell. This circulation resulted from the relative particle to fluid motion and the velocity gradient caused by the different removal rates from the two outlets. Several arrangements of baffles and two passage widths (4 and 12 mm) were tested and the flow cell resulting in the least amount of turbulence was finally chosen. For typical flow velocities, the Reynolds number based on water viscosity was in the range of 10–100.

Individual steel and glass spheres were used for the single particles studies but for these continuous flow studies, quantities of quartz and galena were screened and narrow size ranges were tested at any one time. This enabled the motion of smaller particles to be followed, and hence extended the range of the dimensionless amplitude scale. Preliminary experiments in the closed cell used for the single particle studies indicated that quartz sand ($\rho_p = 2.65 \times 10^3$ kg/m^3) and galena ($\rho_p = 7.8 \times 10^3$ kg/m^3) particles followed basically the same mean paths as glass and steel particles respectively, because of the similarity of their respective densities. There was however a tendency for the sand and galena particles to jump around due to their angularity.

Because of the difficulties associated with feeding quantities of large particles into the continuous flow cell, a vertical amplitude of motion of 1 mm was used for these tests to maintain the amplitude scale for the smaller 500–600 μm particles similar to the value of 1.9 for the tests in figure 3. It was also more acceptable to use water as the continuous medium rather than glycerol solutions as used for the results in figure 3. Preliminary tests were thus conducted with a single 540 μm glass particle in water and the rising motion was observed for a frequency of 94 Hz. These results are presented in figure 7. Although the acceleration number is below the value for the experiments of figure 4, where an appreciable separation tendency was indicated, extrapolation of the results of figure 7 suggests little difference in particle velocity for 94 Hz ($G = 35.6$) and 112 Hz ($G = 50$) so that the basic experiments were carried out at the lower frequency of 94 Hz. This lower frequency was chosen mainly because of mechanical limitations of the experimental rig when run continuously.

An indication of particle relative velocities could be determined by noting whether particles tended to rise, and were removed through to top hole or tended to fall, in which case they would exit through the bottom hole. With water flowing through the test cell and oscillations of (1 mm, 94 Hz) vertical and (4 mm, 47 Hz) horizontal, and phase angle of $-3\pi/8$, a small number of 500–600 μm sand and galena particles were introduced. There was complete separation, with the sand particles migrating to the top exist while the galena particles were removed from the bottom exit.

Dimensional analysis

The single particle studies have verified that the velocity ratio for a particle of given density depends on the Stokes number, amplitude, acceleration number and phase angle. It was shown that the ideal angle to separate particles of different specific gravity is close to $-3\pi/8$, so that this variable was retained constant. The degree of separation of two species of particles with different densities (such as sand and galena) depends on the Stokes number and amplitude for the constant acceleration number chosen for these tests. With the continuous flow system however, other variables must also be considered, including system geometry, flow velocities and solids concentration. No tests were done to consider the effects of system geometry on scaling to various size particles, but some effort has gone into achieving the best types of flow passage within the limitations of the existing rig. The characteristic flow velocity was controlled by removal rates of either or both the tail (top exit) and the concentrate (lower exit) and this is discussed below when the other parameters are considered individually.

Feed concentration

The feed concentration is an important variable affecting the performance of any separating system of the type being investigated here, so this aspect of performance was studied first, with slurries of 500–600 μm sand and galena.

Solids concentrations in the feed of up to 35% by weight were tested and the results in terms of enrichment ratio and recovery are presented in figure 8. These variables are common means of quantifying separation in minerals processing equipment and have been defined in the appendix. In all of these tests, a nominal feed grade of 5% galena by weight (5% of total solids) was employed, although this was difficult to control because of the relatively high settling velocities in the mixer and feed lines. The grades were determined by sampling the flow for a known period and determining the quantity of sand and galena in the concentrate and the tail by heavy media separation (using tetrabromoethane). The results in figure 8 show that the separation performance deteriorated rapidly for concentrations above about 20% solids by weight. This suggests a limit beyond which the particle–particle interaction begins to dominate the relative particle to fluid motion, so that the results of single particle studies are no longer applicable. This limit depends to some extent on particle size as well.

Amplitude scale

The effects of variation in amplitude scale were assessed by experimenting with different size particles, since amplitude scale is the only parameter which includes the variable *d* (particle diameter). A feed concentration of around 20% was maintained (or just below) and an attempt was made to maintain nominal feed grades of around 5%. The results for tests in water ($St \times 10^3 = 1.69$) may be seen in figure 9. The parameter (d/A_V) has been used instead of (A_V/d)

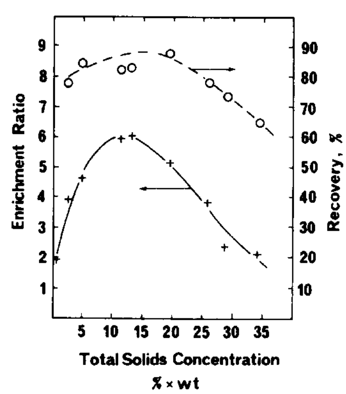

Figure 8. Effect of solids concentration by weight on feed separation performance of continuous flow cell. Slurries contained 500–600 μm screened sand–galena, 5% nominal feed grades. $St \times 10^3 = 1.7$, $G = 35.6$, $A_V = 1$ mm.

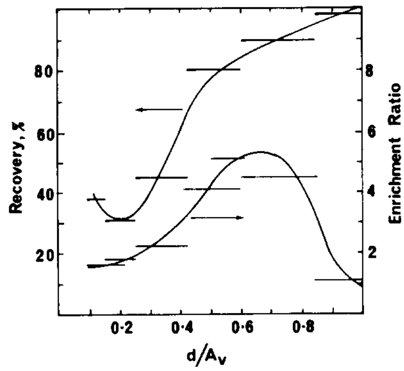

Figure 9. Separation performance as a function of amplitude scale (particle size). $St \times 10^3 = 1.7$, $G = 35.6$, $A_V = 1$ mm. Feed concentration 20% nominal.

to simply relate these results to particle size since the ratio (d/A_V) actually gives particles size in millimetres for these tests. The results are presented in the form of enrichment ratio and recovery. There is a range of particle sizes (d/A_V) for which significant upgrading occurs and this would be e.g. from 420 to 850 μm (a factor of 2 in size).

Stokes number

This parameter was varied independently by changing the viscosity of the liquid. Experiments identical with those of figure 9 were repeated with 12 and 120 cp aqueous glycerol solutions, giving $St \times 10^3 = 17$ and 170 respectively. For the 12 cp glycerol, there was virtually no selectivity of materials at all, while the 120 cp there did seem to be an enrichment of about 2–3 times. However, it was observed that for both of these fluids, both the galena and sand for all size ranges tended to settle on the bottom of the test cell so that any material removed via the tailings exit was simply entrained by the flowing fluid.

Flow rates

The slurry was fed to the separating cell under pressure, so that flow rates were controlled by the back pressures in the settling chambers for the tailings and the concentrate. In all cases the concentrate was removed intermittently by the operator when a significant amount of galena was observed to build up. This obviously introduced some uncertainties into the process and accounts for the fact that enrichment ratios varied from test to test, but the same operator was used throughout in order to achieve some consistency. The tailings were removed continuously and this was the rate controlling factor. For the tests with 12 cp glycerol, and for the 120 cp glycerol, the solids packed into the bottom of the cell and the galena tended to work its way through the sand. For these two fluids, it thus seems that entrainment of the lighter particles by the fast flowing fluid combined with a simple jigging affect was the mechanism which caused any selectivity of materials.

Acceleration number

The experiments discussed above have all been at constant frequency and amplitude, and hence acceleration number $(G = 35.4)$. The single particle studies indicated that decreasing benefit would be obtained from increases in acceleration number. To check this fact experiments similar to those given in figure 9 were repeated, but with an increased frequency so that acceleration number was increased ($G = 50.0$ so that maximum acceleration $= 69 \times$ gravitational acceleration). There was a marginal improvement in performance for the larger particles (with enrichment ratios up to 7.5) but a deterioration in performance for the smaller particles, probably due to an increase in mixing within the cell because of these more intense oscillations.

Discussion

Experiments with the continuous flow cell have verified that a separation of quartz and galena particles will occur for the conditions indicated by the single particle studies. These conditions may be described in terms of the dimensionless parameters:

Stokes number: 1.7×10^{-3};
Amplitude scale: 2.0;
Phase angle: $-3\pi/8$;
Acceleration number: 35.

The experiments suggested that improvement in separation will occur as the acceleration number is increased, but that additional benefits tend to diminish as the acceleration number is further increased.

The single particle studies indicated that a separation could be achieved for values of Stokes number equal to 0.26×10^{-3} and 5.52×10^{-3} (see figure 5) and as shown in figures 8 and 9, separation did occur for Stokes number equal to 1.7×10^{-3}. For significantly higher values however separation was not evident.

For the range of Stokes numbers referred to above, the amplitude scale was also important and best separation was achieved for $1.2 < A_v/d < 2.4$.

In all of these experiments, the ratios of vertical to horizontal frequency and amplitude have been maintained constant. It is most likely that other combinations of these parameters may also cause selective separation and may even result in an improved performance, but it is thought that any improvements would be only marginal. In any case the ranges of the other dimensionless parameters to cause separation would probably be similar to those found above, so that the results of these experiments should serve as an indicator of the range of conditions for which successful particle separation can be achieved.

CONCLUSIONS

The motion of individual particles in a liquid can be controlled by the imposition of combined vertical–horizontal simple harmonic oscillations on the liquid. The particle motion is determined by seven dimensionless variables, five of which were systematically varied in this study. A numerical treatment of the one-dimensional equation of particle motion applied to this situation did not accurately predict mean particle velocities, but it did give qualitative agreement with experiment and predicted the possibility of particles rising against gravity.

The single particle studies indicated conditions at which particles could be separated on the basis of their density differences with an enhancement of their relative velocities when compared to steady-state settling. These studies were extended to slurries containing different density particles, suggesting a potential application of the results in which mineral slurries could be upgraded. Slurries of relatively coarse particles were treated to give concentrations of the heavier material with enrichment ratios above 6 and recoveries approaching 90% at the conditions suggested by the single particle studies, and no doubt these values could be improved by further development of the hydrodynamics of the flow system. It was found that for the range of experiments conducted, three dimensionless parameters were sufficient to characterise the flow and that separations could be achieved within limited ranges of these parameters. The precise limits have not been determined in every case, but as a general guide the ranges are: $0.2 \times 10^{-3} < St < 6 \times 10^{-3}$; $1.2 < A_v/d < 2.4$; $G > 20$.

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APPENDIX

"Goodness" of separation

A frequently used means of representing the "goodness" of separation of a minerals processing operation is by means of the grade and recovery. If the aim is to remove galena concentrate from a quartz–galena slurry, then the definitions are:

$$\text{grade} = \frac{g_c}{g_c + q_c},$$

$$\text{and recovery} = \frac{g_c}{g_c + g_t}$$

where g and q represent the galena and quartz weight flow rates and c and t represent the concentrate and tail respectively.

To allow for variations in the grade of the feed slurry, it is often convenient to use the enrichment ratio, rather than the grade, where

$$\text{enrichment ratio} = \frac{\text{grade of concentrate}}{\text{grade of feed}}.$$